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Immersive Contact Geometry: A VR-Based Approach on Embodied Understanding of Legendrian and Transverse Knots

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Abstract

One of the main challenges to teaching advanced mathematics is understanding high dimensional geometric structures. Students' spatial understanding is limited by contact geometry's dependency on static 2D projections, which includes abstract concepts like Legendrian and transverse knots. The goal of this study is to present an immersive Virtual Reality (VR) environment that allows students to see and engage with three-dimensional contact manifolds and tight vs. overtwisted contact structures in them. To model contact planes and their relationship to Legendrian and transverse knots, a virtual reality learning environment was created by using Gravity Sketch. Participants in a pilot implementation explored knots in both standard and overtwisted contact structures in instructors supervision during a graduate summer school. Observations, informal interviews, and fieldnotes were collected to capture student engagement and conceptual understanding. According to participants, the VR experience allowed them to "see" tangency, cusps, and invariants for the first time, turning abstract concepts into real-world experiences. By using this method, teachers were able to spot misconceptions that were previously undetectable in 2D environments. Even if the sample is small, immersive virtual reality holds great promise for use as a precision pedagogical tool that facilitates embodied and conceptual learning in advanced mathematics, rather than just as a visualization tool. Formal pre/post assessments will be used in future research to measure conceptual advancements and spatial reasoning.

***Keywords*:** Virtual Reality, Contact Geometry, Legendrian and Transverse Knots, Mathematics Education, Spatial Visualization, Human-Computer Interaction

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1. Introduction

In mathematics education, students often struggle to visualize and interpret structures. Textbooks often present geometric shapes as static images, making it difficult to think in three dimensions, which limits students' holistic understanding of geometric concepts [11]. This challenge becomes even more pronounced in advanced topics such as contact geometry, where abstract entities like Legendrian and transverse knots require students to interpret relationships between curves and contact planes. These topics are typically taught using fixed two-dimensional projections, but these representations can lead to misconceptions. Visualization plays a crucial role in understanding geometric concepts, helping students mentally manipulate and explore figures to solve related problems, and leading to a deeper comprehension of the material [17]. In this context, new technologies developing in the field of Human Computer

Interaction are groundbreaking, especially those in the field of virtual reality, playing a transformative role in mathematics education by helping the learners grasp abstract concepts more concretely. Immersive and interactive learning environments offer students innovative mathematical experiences, transforming teaching methodologies and have the potential to increase students' engagement with mathematical concepts in advanced subjects, enhancing students' learning experiences and improving their mathematical skills [3,10]. Virtual reality (VR) technology presents abstract mathematical structures in a three-dimensional, interactive environment. Spatial skills are vital in the fields of geometry and calculus in the acquisition of mathematical knowledge [2, 5, 14]. Numerous studies in the literature demonstrate that VR and AR support geometry and spatial thinking, with findings demonstrating that these concepts enhance students' spatial visualization skills [6].

However, most existing examples focus on more elementary geometric problems; there is very limited research that directly employs VR to address conceptual misconceptions at the level of contact geometry. Existing studies such as Knotted Portals in VR [23] primarily explore branched covers, not contact-geometric structures. Similarly, KnotPlot [20] allows users to visualize and manipulate knots in 3D (within $\mathbb{R}^3$) and can also be experienced in VR, yet it focuses solely on the spatial properties of knots rather than their behavior in a contact manifold. Resources such as the Legendrian Knot Atlas provide 3D visualizations and projections (front and Lagrangian) they are not integrated into a VR setting. The present study fills this gap by developing a VR learning environment that enables students to interact with Legendrian and transverse knots directly within a modeled contact space, making abstract notions such as tangency, cusp formation, and classical invariants explicitly observable in three dimensions. This study investigates how enabling learners to experience geometric structures dynamically through VR can improve comprehension in an abstract field such as contact geometry. The strategy emphasizes fair access to excellent and innovative teaching resources, which is in line with Sustainable Development Goal 4 (Quality Education) of the United Nations (UN). In this study, we propose three research questions:

- How can immersive VR environments support learners' conceptual understanding of abstract geometric constructions, specifically Legendrian and transverse knots in contact geometry?
- In what ways does interacting with a 3D contact space through VR influence learners' spatial visualization and embodied reasoning compared to traditional 2D representations?
- What types of misconceptions about contact geometry can be identified and addressed through immersive VR-based exploration?

2. Methodology

2.1 Research Design

This study used a qualitative exploratory design within a Design-Based Research (DBR) framework [8]. The DBR approach was selected because the goal was not to test a set hypothesis. Instead, we aimed to design, implement, and evaluate a intervention that combines immersive visualization with mathematical thinking.

2.2 Participants and Context

A pilot implementation was conducted during a graduate-level summer school on introduction to geometric topology organized by Turkish Women Mathematicians Association in Ankara with the participation of 23 graduate students (20 female; 3 male; aged 21-26) and four instructors who are specializing in mathematics and educational technology. The pilot work done in the eight day of the summer school, all participants had taken courses which are based on fundamental concepts of geometric topology. Also before the implementation of VR theoretical lecture related to knot theory and Legendrian and transverse knots and their invariants was covered. All participants took the course but had not used VR based learning tools before. Participation was voluntary, and we obtained informed consent according to university ethical guidelines.

2.3 VR Environment Design

We developed the immersive environment using Gravity Sketch, a VR application designed for use with the Meta Quest 2 headset. We chose this software for its ability to let learners draw, rotate, and scale mathematical objects in 3D using natural hand movements.

Two main contact structure configurations were modeled (Figure 1, Figure 2):

The tight contact structure environment illustrates the standard contact distribution as the kernel of the one form dz-ydx in $\mathbb{R}^3$ which allows the user to see how to draw a Legendrian curve which must be tangent to contact planes versus a transverse curve which must be transverse to contact planes.

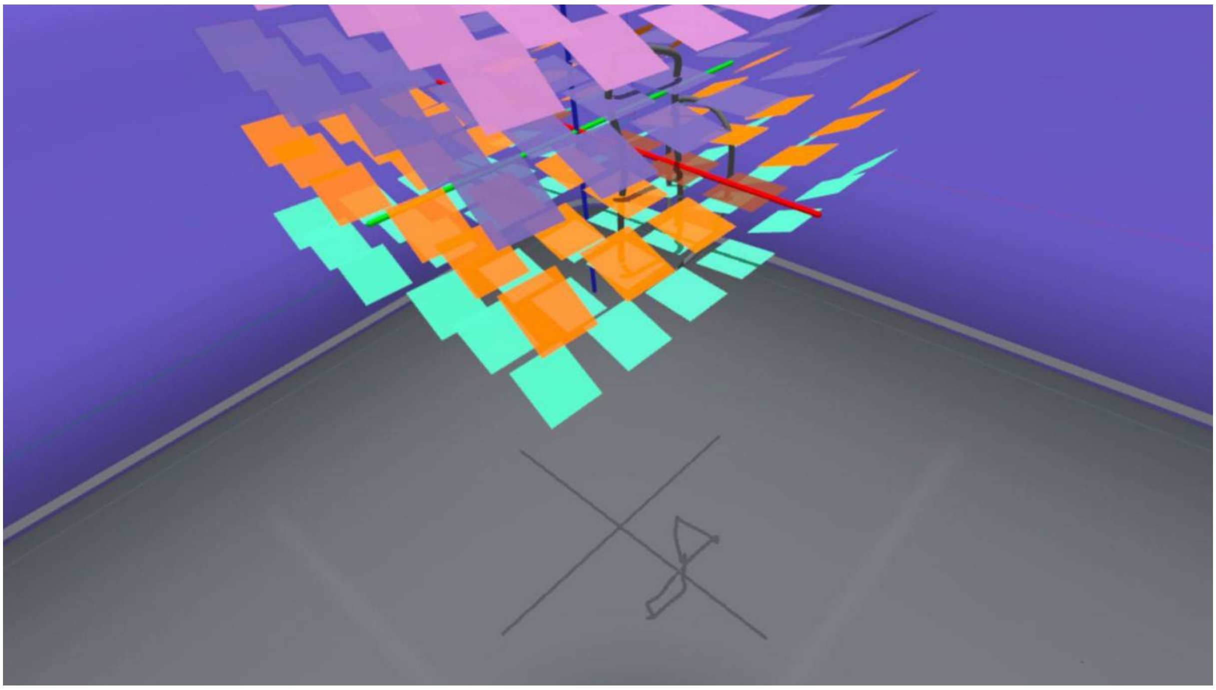

***Figure 1**. Visualization of a transverse knot within the standard tight contact structure.*

The overtwisted contact structure environment in $\mathbb{R}^3$ as the kernel of the one form $dz+r^2d\theta$ which featured local embedded "overtwisted disk" that visually demonstrate contact planes twist at least 180 degrees. Along its boundary all planes are horizontal, so they are tangent to the boundary circle of the overtwisted disk. Therefore, it provides an explicit example of a Legendrian unknot. One can also see its front projection.

VR environments are useful to clarify the differences between tight and overtwisted structures. From the definitions of both contact plane distributions, there must be a plane at any point in space, but in reality it is not possible to draw each plane. So, while constructing these environments, as it is done in the existing literature, we draw finitely many particular planes that is enough to make the twisting understandable.

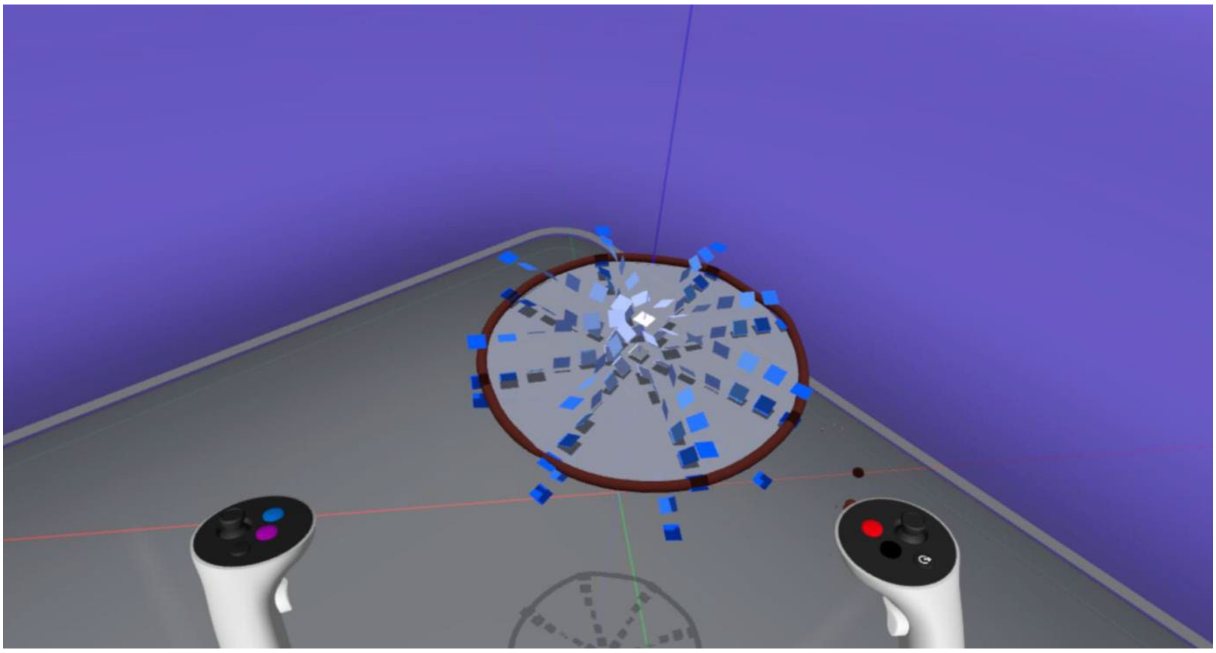

***Figure 2**. Overtwisted disk representation and front projection of its Legendrian boundary in the designed VR environment.*

Each structure had interactive contact planes, color-coded to show their orientation relative to the knot. Learners could move through these planes and observe how the projections changed along the knot. This design supported embodied cognition by connecting physical movement with visual feedback for better understanding. Before deployment, the environment was improved through two rounds of usability tests with instructors.

2.4 Procedure

Theoretical Preparation: Participants in the pilot study had previously attended a two-hour contact geometry lecture that introduced essential concepts such as knots, their invariants, and the distinction between Legendrian and transverse knots. The lecture also addressed the theoretical framework of contact structures and their corresponding invariants, as a preliminary orientation, this session allowed participants to build the conceptual foundation for the immersive experience. Following this theoretical foundation, a one-hour practical session began.

The experimental session lasted about 60 minutes and included three phases:

Orientation (10 minutes):

Instructors presented the VR equipment setup and described how the immersive environment would visualize the previously discussed mathematical structures during the pilot session, which started after this preparatory lecture in a more casual, interpersonal setting. Before entering the virtual environment, this transition phase gave participants the chance to express their expectations, ask insightful questions, and get familiar with the headset interface. Instructors demonstrated how to navigate within the VR environment by moving through contact planes and verbally explaining each step while using the headset. This process served both as a technical orientation and a conceptual bridge, reinforcing the link between theoretical content and its three-dimensional visualization.

Immersive Exploration (40 minutes): Each participant who is voluntary, used the headset individually under supervision. Instructors provided prompts like, “Where do you see tangency?”, “Can you draw a surface where these planes are tangent to all of the surface? or “Can you see how much the planes rotated, is this knot Legendrian or transverse, how their projections look like?” Learners created knots, intersected them with contact planes, and moved around the 3D space to observe geometric structures.

Reflection and Discussion (10 minutes): After the session, participants discussed their thoughts, instructors noted observations and conducted brief interviews. The reflections focused on conceptual insights, engagement, and emotional responses to the immersive experience. We recorded all interactions for qualitative analysis (Fifure 3).

Figure 3. *Participant exploring tight contact structures in the VR environment developed in Gravity Sketch using Meta Quest 2.*

2.5 Data Collection Instruments

Data collected through two qualitative data collection tools. Structured observation sheets recorded learners' navigation strategies, error frequency, and spontaneous comments that indicated understanding or confusion. And semi-structured interviews which is held right after the session to gather emotional and cognitive responses (e.g., perceived clarity, engagement, spatial insight). Additionally, instructor reflections provided expert validation. For example, one instructor noted that "many expressed that this was the moment they truly grasped the underlying structure of these concepts, an understanding that became tangible only through the VR experience. This work underscores the growing importance of VR technologies in mathematical communication and education, particularly for abstract geometric and topological concepts." confirming the alignment between immersive experience and conceptual learning goals. Using multiple sources helped ensure reliable interpretations [9].

2.6 Data Analysis

Data will be analyzed using thematic analysis [7]. Codes will be generated inductively from observation notes and interview transcripts to identify recurring themes related to:

- Conceptual understanding,
- Spatial reasoning, and
- Engagement and motivation.

Two researchers will conduct independent coding and will later compare the results to ensure interpretive reliability through peer debriefing.

Triangulation across observations, interviews, and instructor notes will strengthen the credibility and validity of the findings.

2.7 Ethical and Accessibility Considerations

All procedures followed the university's ethical research guidelines. Participation was voluntary, data were anonymized, and participants could leave at any time. We took precautions to minimize discomfort and motion sickness. We also ensured accessibility by providing equal access to headsets, clear visual designs, and assistance during navigation. These considerations support the principles of inclusive and sustainable teaching under SDG 4 (Quality Education), focusing on fairness, inclusion, and responsible use of new educational technologies.

3. Results and Discussion

3.1 Preliminary Observations

As the formal data analysis has not yet been conducted, this section summarizes the initial observations and reflections gathered during the pilot study.

Throughout the VR sessions, all 23 participants showed active engagement and curiosity. They observed the immersive environment, interacted with Legendrian and transverse knots, and explored the orientation of contact planes. The instructors noticed that participants were not only attentive but also visibly interested in the geometric behavior of the structures. They frequently voiced their thoughts or asked questions during the exploration.

One notable moment involved a misunderstanding about cusps in Legendrian projections. A participant initially thought that the cusp points indicated breaks in the knot or interruptions in its smoothness. However, after manipulating the model and seeing the continuous nature of the contact planes through the cusp region, the student realized that the cusp is a projection artifact, not a geometric break. This realization happened spontaneously through direct interaction instead of instructor explanation. It highlighted how embodied visualization can challenge and correct abstract misunderstandings.

3.2 Instructor Reflections and Pedagogical Insights

Instructor reflections provided valuable insights into these informal findings. One instructor noted that "many students expressed that this was the first time they truly grasped the underlying structure of these concepts; the visualization made the idea of tangency clear." Another commented that "the VR activity changed the understanding of Legendrian and transverse knots in tight and overtwisted contact spaces, which is normally abstract and hard to imagine, into visual and spatial experiences."

These insights suggest that immersive visualization may help link symbolic abstraction and spatial intuition. Learners can see and manipulate properties that are usually confined to theoretical representation. The embodied experience supported conceptual grounding, an observation consistent with previous findings on spatial thinking in immersive learning [1].

3.3 Engagement and Feasibility

The activity maintained a high level of engagement. Participants showed enthusiasm and collaboration, often helping each other, providing guidance on what their peer should do when they were observing. The technical performance of the VR environment was stable throughout, and the Gravity Sketch interface was intuitive even for first-time users. Minor issues, like headset calibration and orientation at the start of sessions, were quickly resolved by instructors.

Overall, the pilot confirmed that conducting immersive mathematical visualization sessions is feasible in an academic setting. Instructors emphasized the importance of guided facilitation, where conceptual prompts during exploration helped participants connect their visual experiences to formal definitions. This is a crucial element of learning abstract mathematical concepts.

3.4 Discussion

The initial findings from this pilot study show the potential of immersive VR environments to improve learners' understanding of abstract geometric structures in contact geometry. While formal data analysis has yet to be conducted, classroom observations, instructor reflections, and participant feedback indicate several encouraging trends that match recent research on immersive learning in mathematics.

Firstly, participants seemed to gain a clearer spatial understanding of contact planes and knot behavior when interacting with 3D representations. This suggests that VR might help connect symbolic definitions with visual experiences. Prior studies that have noted the similar results for instance, Su et al. (2022) found that VR greatly improved students' ability to reason about geometric relationships, and Medina-Herrera et al. (2024) demonstrated that immersive visualization boosts learners' spatial intuition in higher-level mathematics. The fact that participants in our study could identify changes in plane orientation, notice tangency, and compare Legendrian to transverse configurations shows that immersive representations might support conceptual understanding even in advanced topics like contact geometry.

Secondly, the pilot uncovered a misconception regarding the interpretation of cusps in front projections. One participant initially thought that the cusp indicated a break or discontinuity in the knot. This error is common in geometry education, where students misread projection artifacts as structural features. The participant corrected the misconception by using direct spatial reasoning to identify the continuous nature of the knot around the cusp through VR-based exploration. This is consistent with previous research demonstrating that interactive 3D environments aid in improving mental models and minimizing projection-based misunderstandings [15, 21]. In fields like contact geometry, where the limitations of 2D diagrams frequently obscure important structural information, VR's capacity to externalize and rectify such misconceptions is especially beneficial.

High levels of interest, engagement, and positive emotional reactions were evoked by the immersive session. Participants seemed inspired to experiment with different knot structures and expressed excitement while navigating the virtual reality environment. Consistent with the literature, this research found that VR encourages long-term interest and enjoyment in mathematics learning. Consistent with the literature, this research found that VR encourages long-term interest and enjoyment in mathematics learning. VR tools inspire students to explore, encourage them to try new things, and carry on the difficult tasks [17, 19]. Since motivation and confidence have a significant impact on conceptual persistence, such affective responses hold an important place in advanced mathematics.

When combined, these early findings imply that immersive virtual reality settings could enhance contact geometry learning in both cognitive and affective domains. They support the expanding corpus of research establishing VR as a potent instrument for giving abstract mathematical concepts a concrete, visually significant, and experientially accessible form. However, the pilot also brought to light the necessity of guided prompts, structured scaffolding, and careful integration with formal mathematical reasoning—a point that has also been made in recent immersive learning studies.

Future analyses that include thematic coding, participant interviews, and assessments of conceptual understanding will better clarify how much VR contributes to measurable gains in understanding, corrections of misconceptions, and improvements in spatial reasoning. As the next phase of this research broadens participant groups and enhances methodological triangulation, these insights will lead to a deeper understanding of how Human-Computer Interaction can enhance high-level mathematical learning in sustainable and scalable ways.

4. Conclusion

This study introduced an immersive VR environment to help learners better understand contact geometry, focusing on Legendrian and transverse knots. The pilot implementation showed that immersive visualization can make abstract mathematical structures more accessible, engaging, and meaningful.

Through interaction with dynamic 3D models, learners could explore the behavior of contact planes in ways that traditional 2D diagrams cannot offer. This experience allowed participants to confront and correct misconceptions, such as misinterpreting cusps, through direct spatial reasoning.

While formal data analysis is still underway, initial observations and instructor feedback suggest that immersive visualization encourages conceptual insight, engagement, and curiosity. The setup's feasibility and the positive response from participants indicate that VR can effectively enhance advanced mathematics instruction alongside symbolic and analytical methods.

Future work will build on this pilot by adding systematic data analysis, quantitative assessment tools, and larger participant groups. These steps aim to provide stronger evidence on how immersive learning environments based on human-computer interaction affect conceptual change and spatial understanding. This research ultimately supports the broader goal of improving sustainable and high-quality education by showing how human-computer interaction can connect abstract theory with hands-on learning in mathematics.

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Authors' Declaration

The authors declare no conflicts of interest

Authors' Contribution Statement

Dr. Elif Medetoğulları conceptualized the study, reviewed the literature and coordinated the implementation process during the graduate summer school. She guided the integration of contact geometry content into the VR models.
Sevim Berhiv Acay designed the immersive Virtual Reality environment, contributed to the pedagogical design, literature synthesis, and methodological development of the study.
Both authors collaboratively refined the structure, data collection through classroom observations and informal interviews, organized instructor reflections, analysis plan, and final version of the manuscript.

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